



Simulation of transverse multi-bunch instabilities of proton beams in LHC

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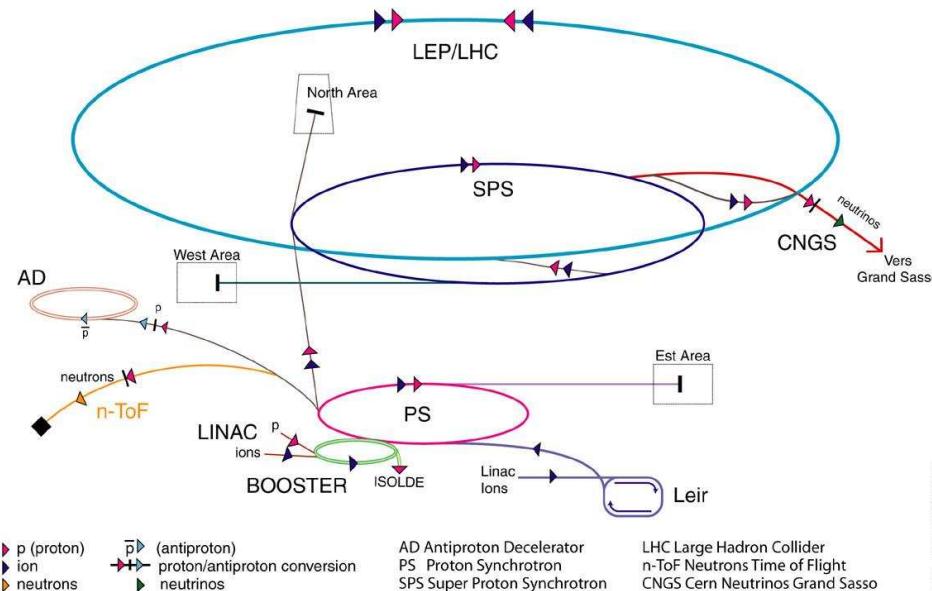
Outline



- Introduction & Motivation
- Simulation Techniques & Approximations
- Resistive Wall Impedance Models
- Measurements in CERN SPS & Comparison to Simulation
- LHC simulated
- Summary

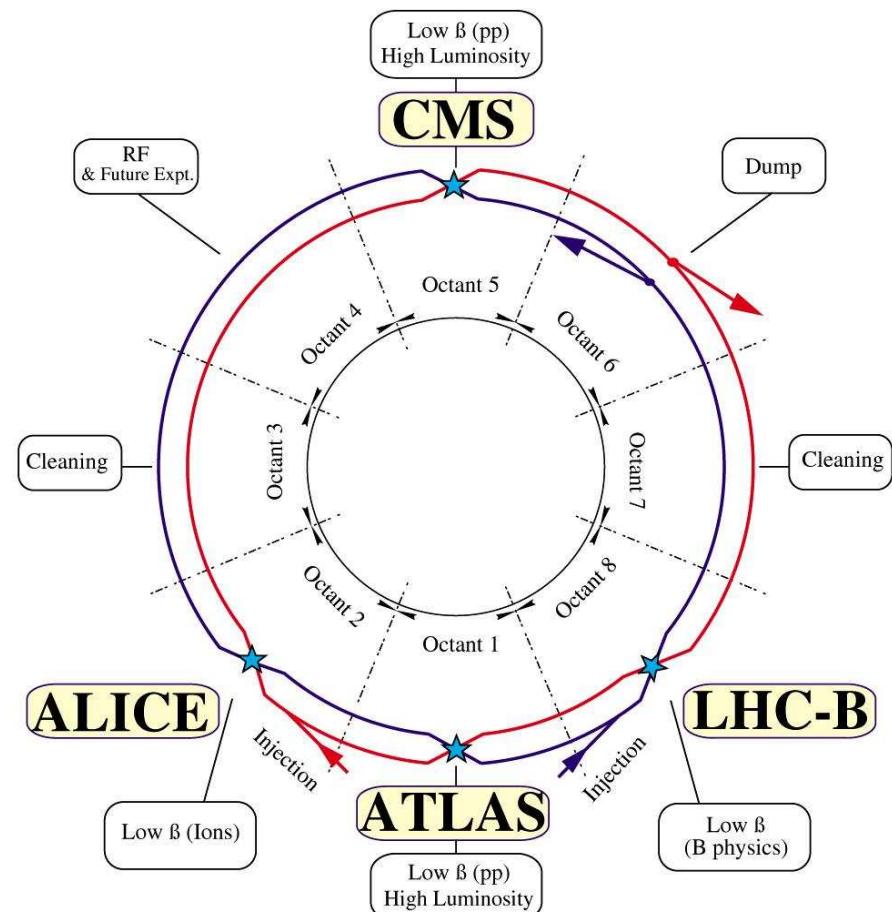


Introduction



Proton-proton collisions with center-of-mass energy of 14 TeV. 2808 bunches per beam.

CERN currently builds the Large Hadron Collider (LHC) in the old 27km long LEP tunnel. Foreseen startup in 2007.



Motivation



Physics demands:
High Event Rate ($N_{\text{Event}} = L \cdot \sigma_{\text{Event}}$) + High Energy (7 TeV)

- High Single Bunch Intensity
- Multiple Bunches
- High Bunch Repetition Rate

$$L = \frac{N_p^2 n_b f_0 \gamma_r}{4 \pi \varepsilon_n \beta^*}$$

Large Luminosity

- Hadrons
- Strong Bending Magnetic Fields needed
 - Superconducting Magnets
 - Small Structure Sizes
- Machine Protection needed (Collimators)

Resulting Problems:
Instabilities

-
- No Radiation Damping
 - Collective Effects
 - Multi-Bunch Effects
 - Multi-Turn Effects
 - Long-Range



Motivation



Stability analysis normally done in frequency domain.

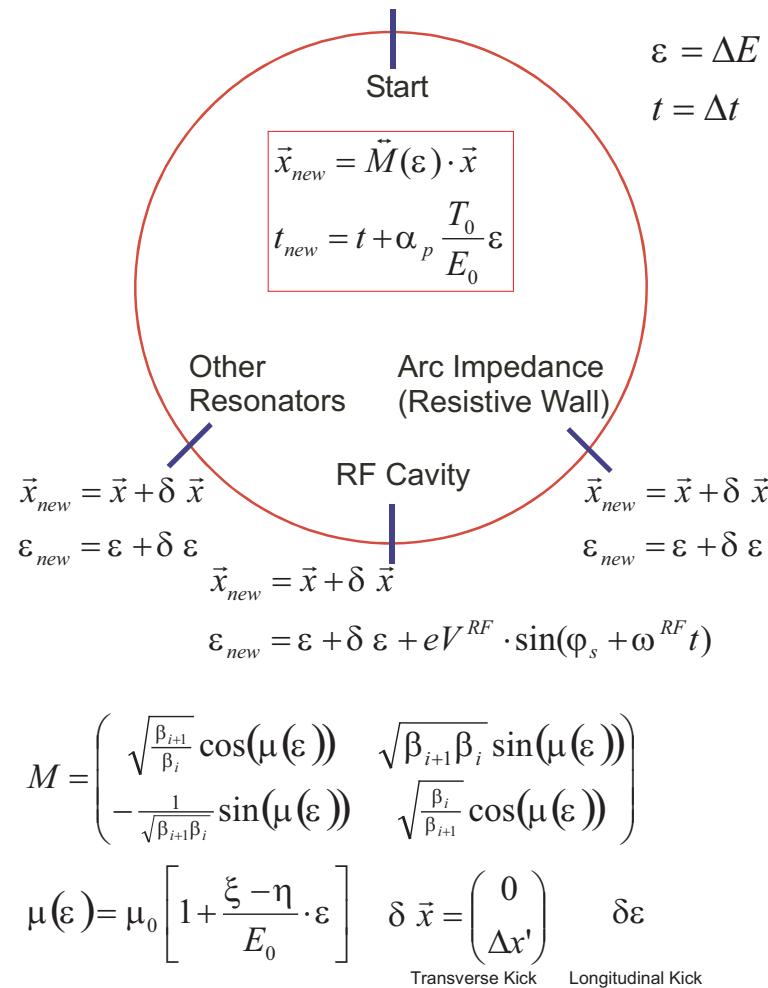
Simulation additionally allows:

- Non-equidistant filling schemes
- Investigate Transition Effects
- Interplay between different effects (impedances)

-
- Long-Range Effects
 - Correct and efficient implementation of corresponding impedances



The Simulation



“Classical” Tracking Code
Linear Transfer Matrices + Kicks

$$\varepsilon = \Delta E$$

$$t = \Delta t$$

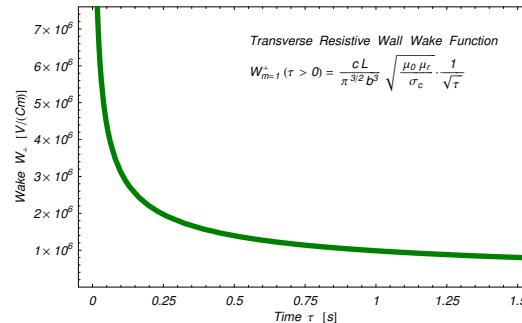
$$\tau \gg \sigma_\tau$$

Long-Range regime: $\sigma_\tau \dots$ bunch length

$\tau \dots$ time interval for wake calc.

Impedances with correspondingly long-lasting wake fields:

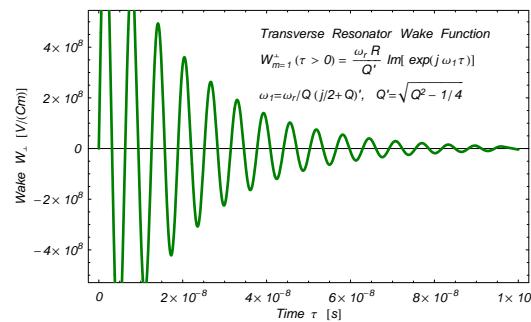
- ➊ Resistive Wall Impedance (with ‘inductive bypass’)
- ➋ Narrow-Band Impedances
(HOMs of cavities, wakes of cavity-like structures)



$$W_\perp(\tau) \propto 1/\sqrt{\tau}$$

$$W_\perp(\tau + \Delta\tau) = ? \cdot W_\perp(\tau)$$

⇒ Fast summation via
FFT Convolution



$$W_\perp(\tau) \propto \exp(j \omega_1 \tau)$$

$$W_\perp(\tau + \Delta\tau) = \exp(j \omega_1 \Delta\tau) \cdot W_\perp(\tau)$$

⇒ Time evolution and summation
using Phasors

= Resonator Model (R, Q, ω_r)

The Simulation / Approximations



- ➊ 1 Detailed Bunch
- ➋ Multiple Bunches represented by 1 super-particle

rigid bunch approximation

$$W_{pot}(\tau) \approx W(\tau)$$

wake function approximation

$$W_{pot}^{\parallel}(\tau) = \int_0^{\infty} dt \ W_{\parallel}(t) \ \lambda(\tau - t) \qquad \qquad \qquad \approx W_{\parallel}(\tau) \text{ for } \tau \gg \sigma_t$$

$$W_{pot}^{\perp}(\tau) = \frac{1}{\bar{\xi}} \int_0^{\infty} dt \ W_{\perp}(t) \ \xi(\tau - t) \ \lambda(\tau - t) \qquad \qquad \qquad \approx W_{\perp}(\tau) \text{ for } \tau \gg \sigma_t$$

Impedance modelled by 1 kick per turn^a
lumped impedance approximation

^a

K. Thompson and R. D. Ruth. *Transverse coupled bunch instabilities in damping rings of high-energy linear colliders*. Phys. Rev., D43:3049-3062, 1991.



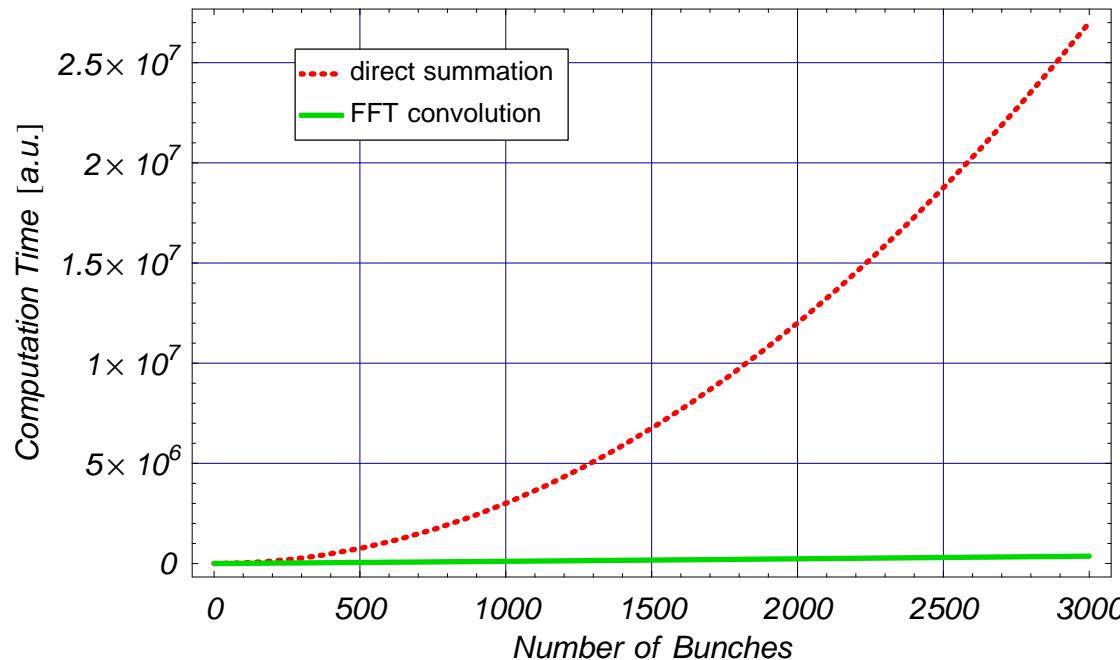
The Simulation / Wake Summation Problem



The kick $\Delta x'_n$ on bunch j at turn n , in the case of the resistive wall impedance:

$$\Delta x'^j_n = \underbrace{\sum_{i=0}^{j-1} \frac{\langle x \rangle_n^i}{\sqrt{(j-i) \cdot \tau_{buc.}}}}_{\text{sum over preceding bunches at current turn}} + \underbrace{\sum_{k=n-n_{mem}}^{n-1} \sum_{i=0}^{N_b-1} \frac{\langle x \rangle_k^i}{\sqrt{(n-k) \cdot \tau_{rev.} + (j-i) \cdot \tau_{buc.}}}}_{\text{sum over preceding bunches over previous turns}}$$

$\mathcal{O}(N_b^2)$



FFT Convolution



Analogy between wake sum and (discrete) convolution:

$$\Delta x'_n = \sum_{k=0}^{n-1} \frac{\langle x \rangle_k}{\sqrt{(n-k) \cdot \tau_{rev}}} = \sum_{k=0}^{n-1} g(k) \cdot f(n-k) \quad (\text{single bunch case})$$

Continuous Convolution
$$h(t) = g * f \equiv \int_{-\infty}^{\infty} d\tau \ g(\tau) f(t - \tau)$$

Discrete Convolution
$$h(n) = g * f = \sum_{k=0}^{N-1} g(k) f(n - k) = \sum_{k=0}^{N-1} g_k f_{n-k}$$

Convolution Theorem
$$\mathcal{F}[f * g] = \mathcal{F}[f]\mathcal{F}[g] \quad \text{or} \quad f * g = \mathcal{F}^{-1}[\mathcal{F}[f]\mathcal{F}[g]]$$

Compute wake sum via the FFT convolution in frequency domain:

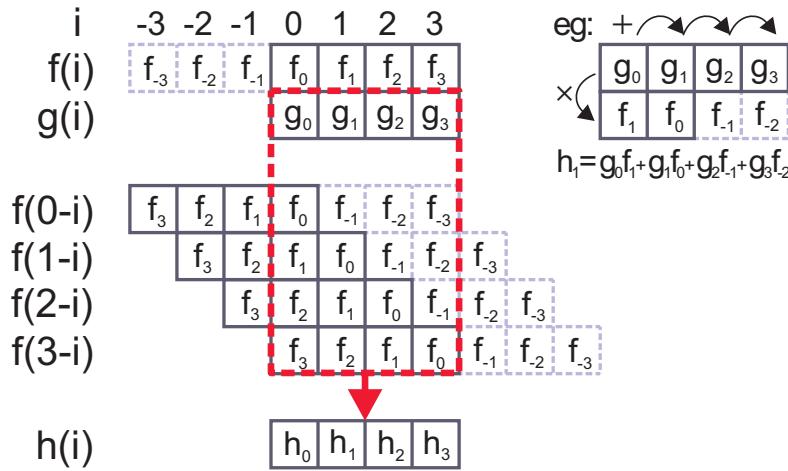
$$\Delta x'_n = h_n = (g * f)_n = \sum_{k=0}^{N-1} g_k f_{n-k} \quad \xrightleftharpoons[IFT]{FFT} \quad G_j F_j = H_j$$



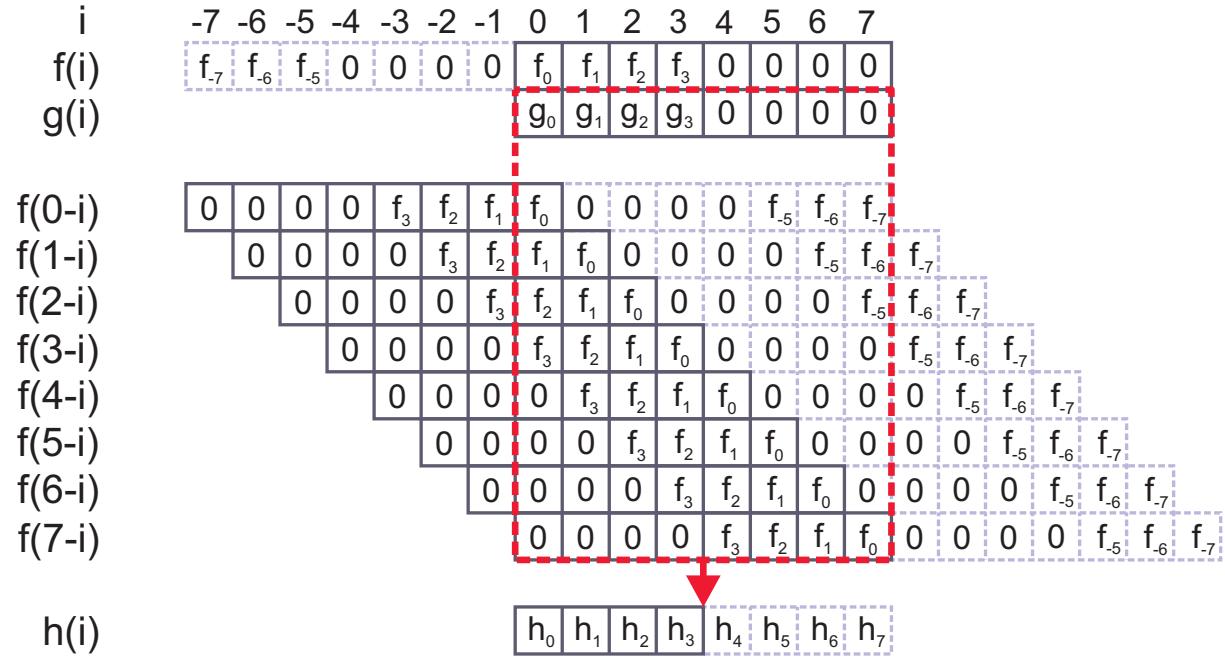
FFT Convolution



Circular Convolution



Linear Convolution



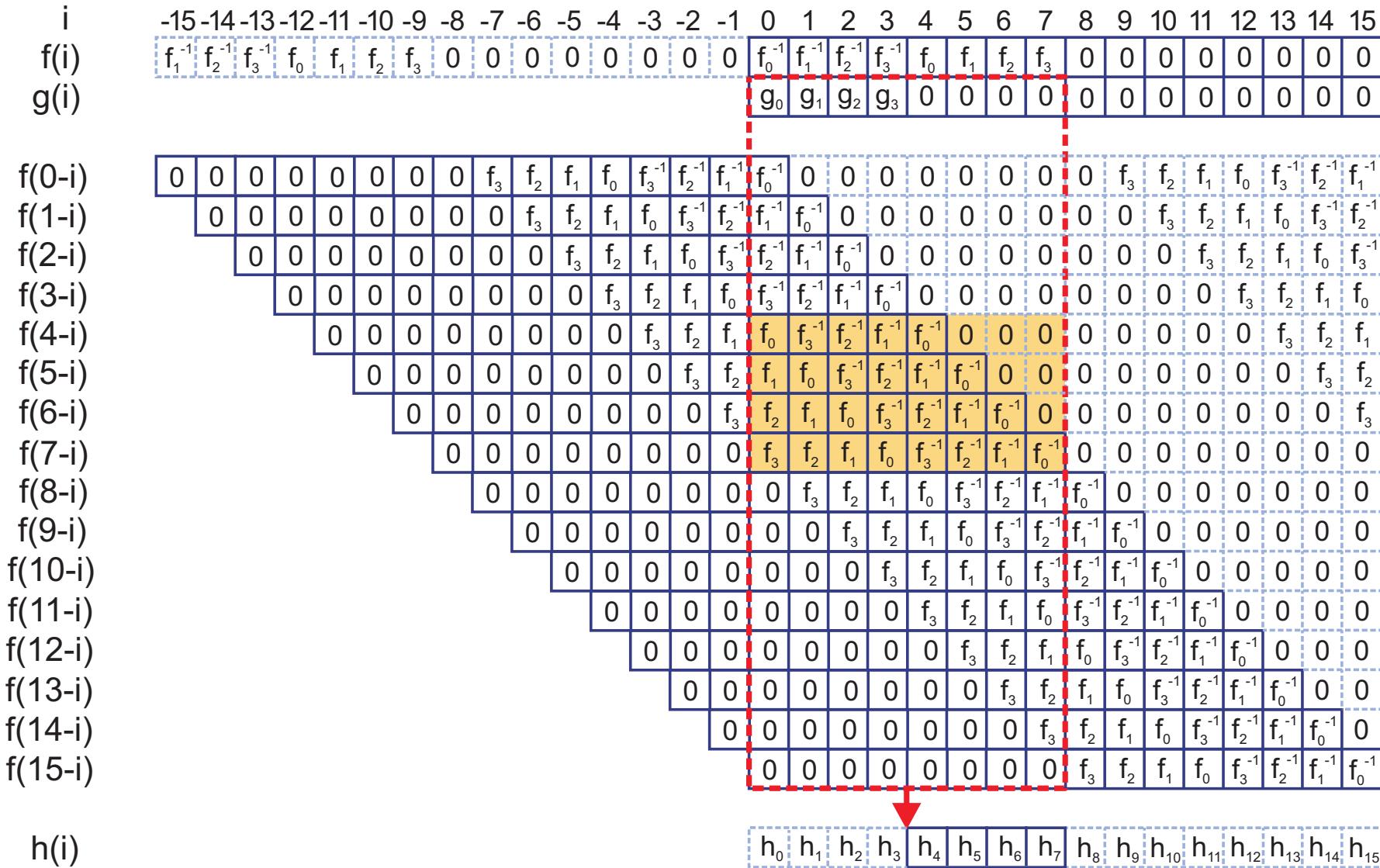
To get the correct wake sums, linear convolution has to be realized by zero padding.



FFT Convolution



Multi-Bunch Multi-Turn Convolution Scheme



FFT Convolution

Wake Summation Algorithm

Search GCD (greatest common divisor) of given bucket layout. Defines new, equidistant bunch pattern. mask.

Set up extended zero-padded arrays f'^c .

Precalculate FFTs of f^c for $c = 0, \dots, n_{\text{mem}}$, this gives arrays $F^c = \mathcal{F}[f^c]$

At turn n do the following:

Write (signal, offsets) to array g_i use mask.

FFT of g , $G^{c=0} = \mathcal{F}[g]$

Multiply $\forall c \in [0, n_{\text{mem}}] : H^c = F^c \cdot G^c$

Inv. FFT $\forall c \in [0, n_{\text{mem}}] : h^c = \mathcal{F}^{-1}[H^c]$

Sum over n_{mem} turns to get the kicks, use the mask: $\Delta x'^j_n = \sum_{c=1}^{n_{\text{mem}}} h^c_{j+N_b} + h^{c=0}_j$.

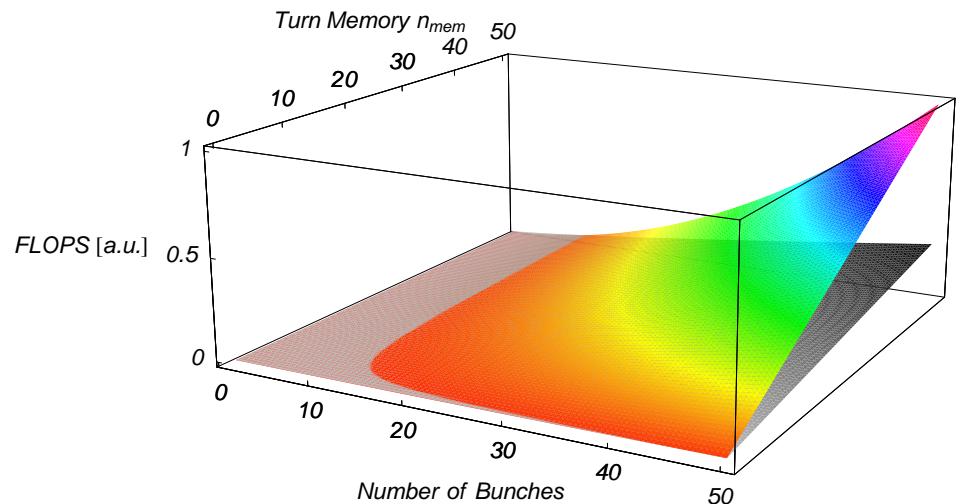
Speed Considerations

Direct Summation

$$(2n_{\text{mem}} + 1) N_b^2 + N_b$$

FFT Convolution

$$(n_{\text{mem}} + 2) 4N_b \log 4N_b + (5n_{\text{mem}} + 5) N_b$$



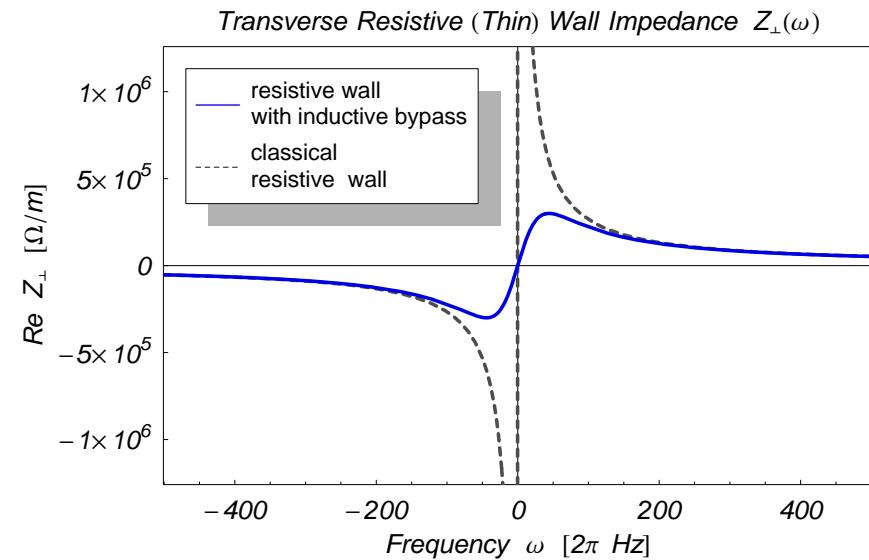
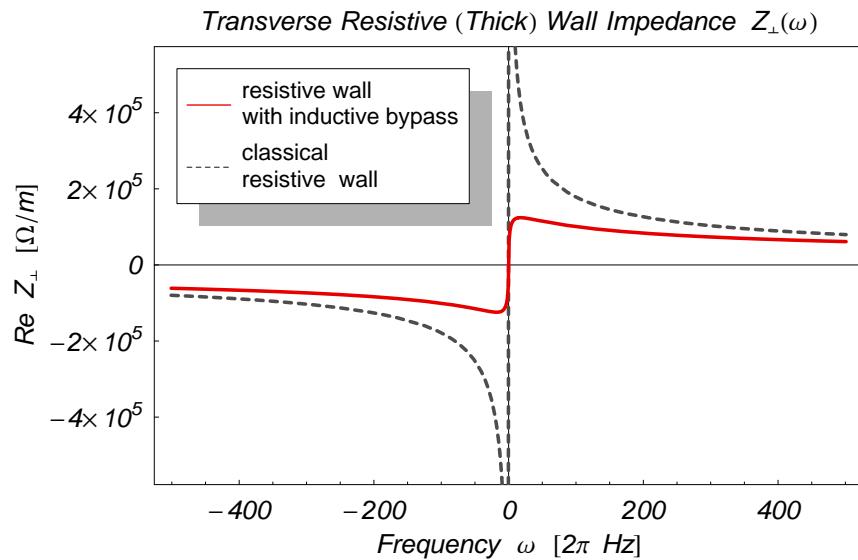
Resistive Wall Impedance Models



Classical thick & thin wall formula known to be incorrect for $\omega \rightarrow 0$.

$$Z_{m=1}^{\perp, \text{thick}}(\omega) = (\operatorname{sgn} \omega + j) \frac{Z_0 L \delta_0 \mu_r}{2 \pi b^3} \cdot \sqrt{\frac{\omega_0}{|\omega|}}$$

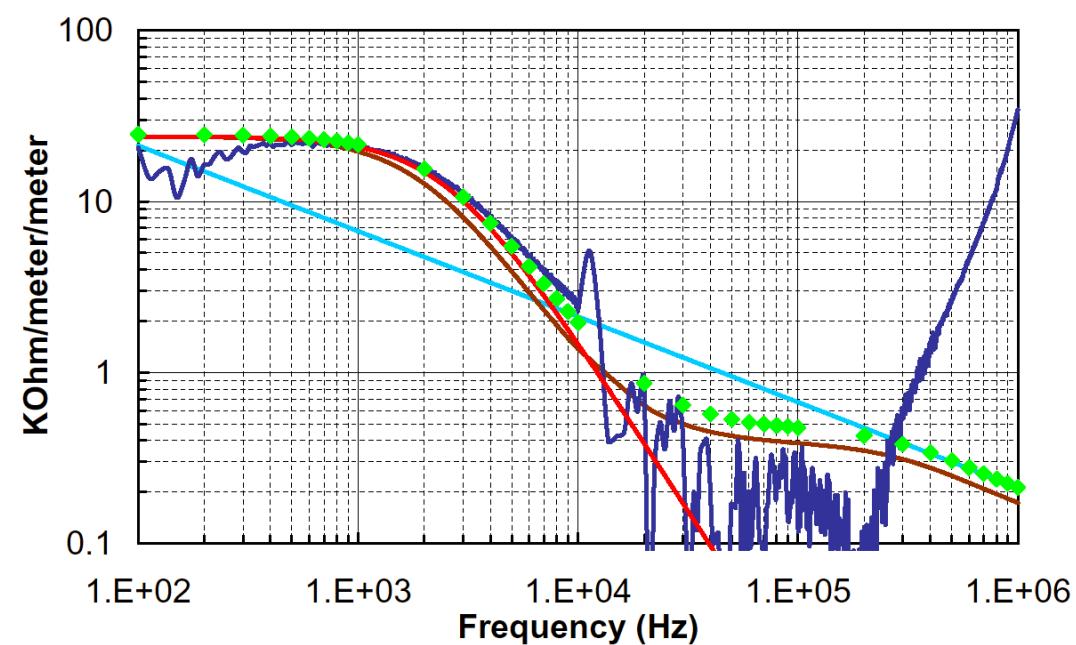
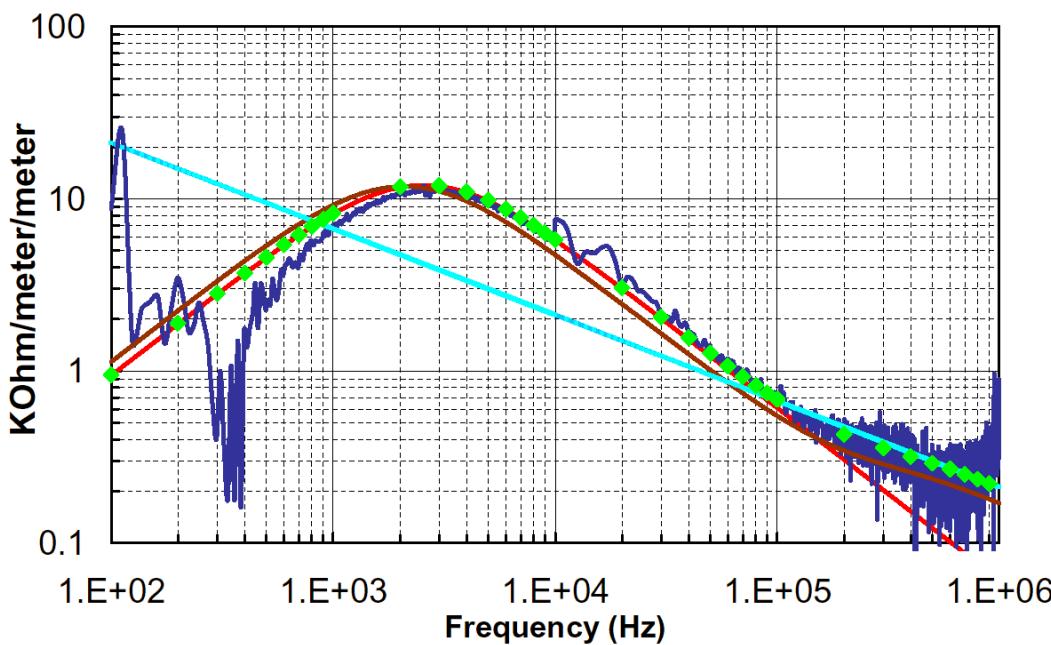
$$Z_{m=1}^{\perp, \text{thin}}(\omega) = \frac{c L}{\pi b^3 \sigma_c d \cdot \omega}$$



Resistive Wall Impedance Models



Measured^a
Transverse Resistive Wall Impedance
Real (left) and Imaginary (right) part



^a

A. Mostacci, F. Caspers, and U. Iriso. *Bench measurements of low frequency transverse impedance*. CERN-AB-2003-051-RF. Proc. of PAC 03, Portland, Oregon, 12-16 May 2003.

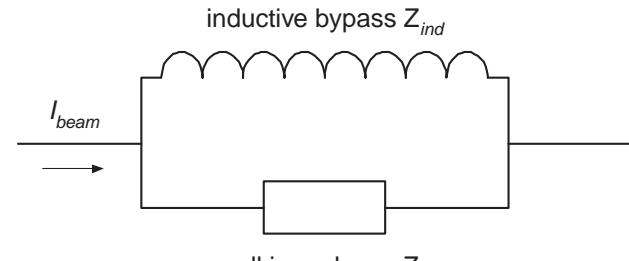


Resistive Wall Impedance Models



Resistive Wall Impedance with 'inductive bypass' (L.Vos)

$$Z_{m=1}^{\perp, LV}(\omega) = \frac{Z_0 L}{2\pi b^2} \cdot \left[\frac{b\omega\mu_0}{2Z_0} \cdot \frac{1 + \frac{Z_0}{Z_1} \tanh \gamma_1 d_1}{1 + \frac{Z_1}{Z_0} \tanh \gamma_1 d_1} - j \right]^{-1}$$



$$Z_{\perp}^{ibp} = \frac{2c}{b^2\omega} \cdot \frac{Z_{\parallel} Z_{\text{ind}}}{Z_{\parallel} + Z_{\text{ind}}}$$

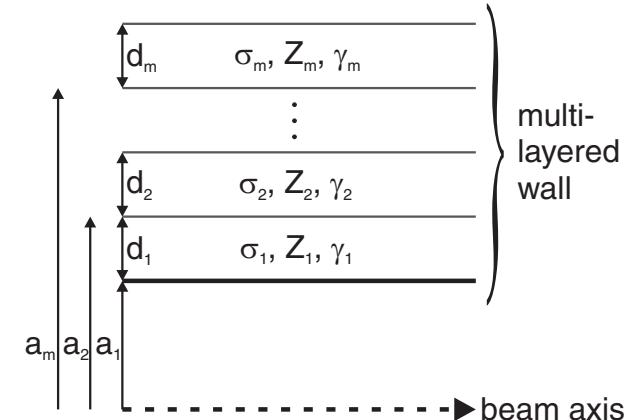
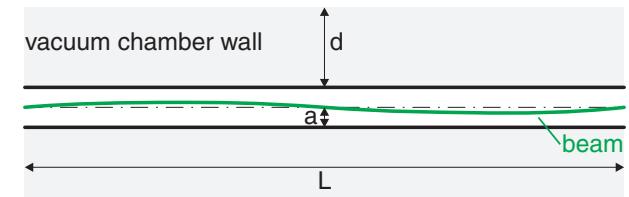
Quasi-Static Beam Model (Burov/Lebedev)

Solving Maxwell Equations, Poisson equation for electric dipole and vector potential for magnetic dipole

$$Z_{m=1}^{\perp, BL}(\omega) = j \frac{Z_0 \beta L}{\pi b^2} \frac{s'_1 + \tilde{\kappa}_{2,1} s_1}{s'_1 + \tilde{\kappa}_{2,1} \tilde{\kappa}_{1,0} c_1 + \tilde{\kappa}_{2,1} s_1 + \tilde{\kappa}_{1,0} c'_1}$$

Field Matching (B.Zotter)

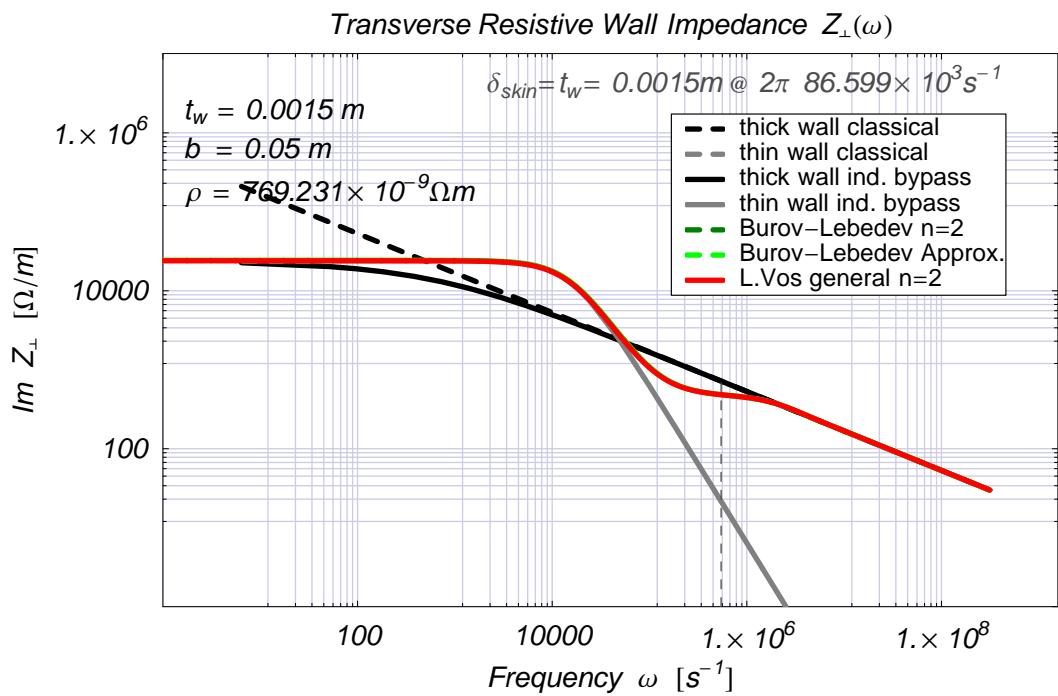
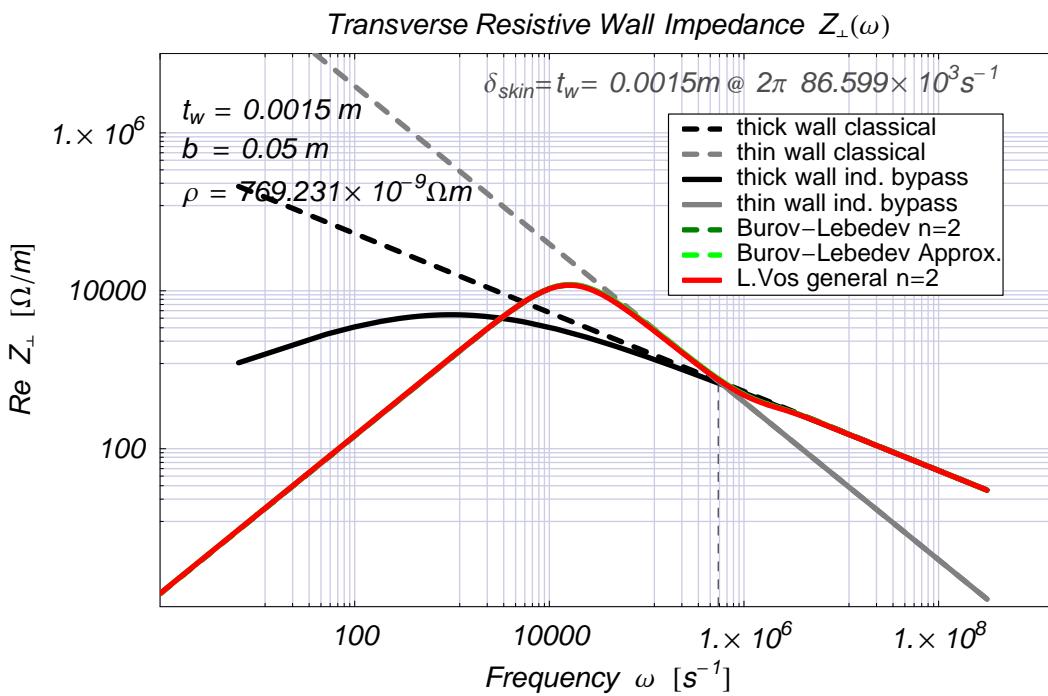
Solution of Maxwell's equation by matching 4 components at every layer. Most rigorous but lengthy or only numeric expressions.



Resistive Wall Impedance Models



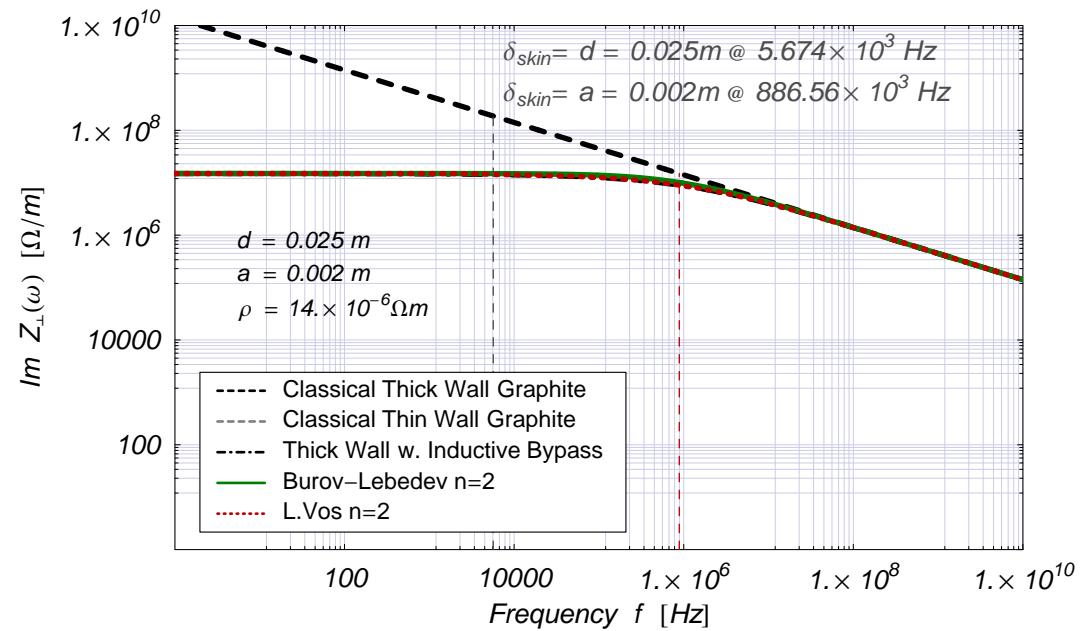
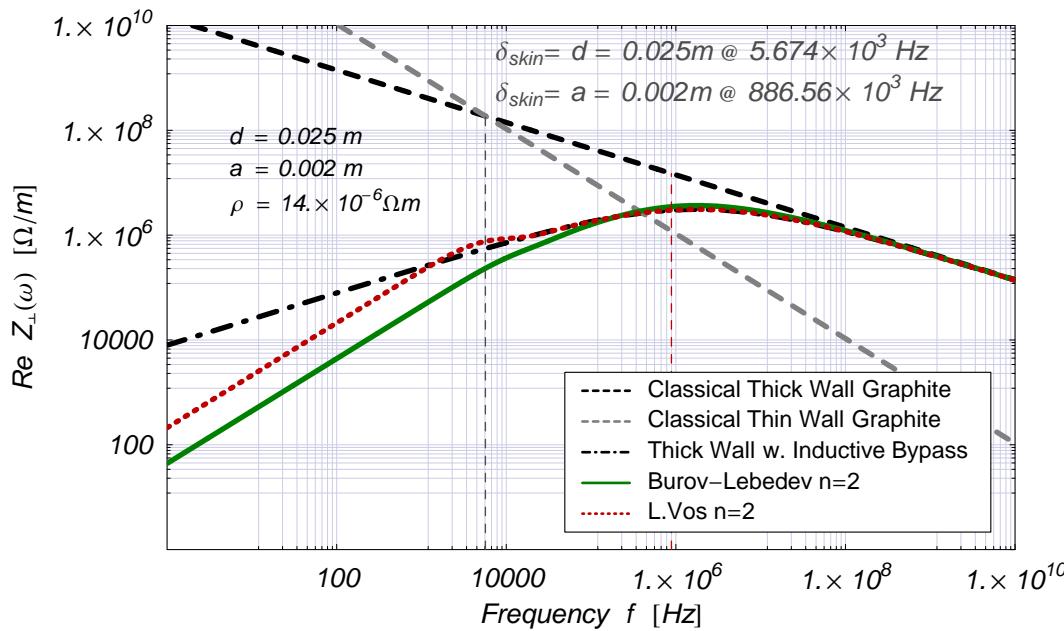
Standard Parameter Range: $b \gg d$



Resistive Wall Impedance Models



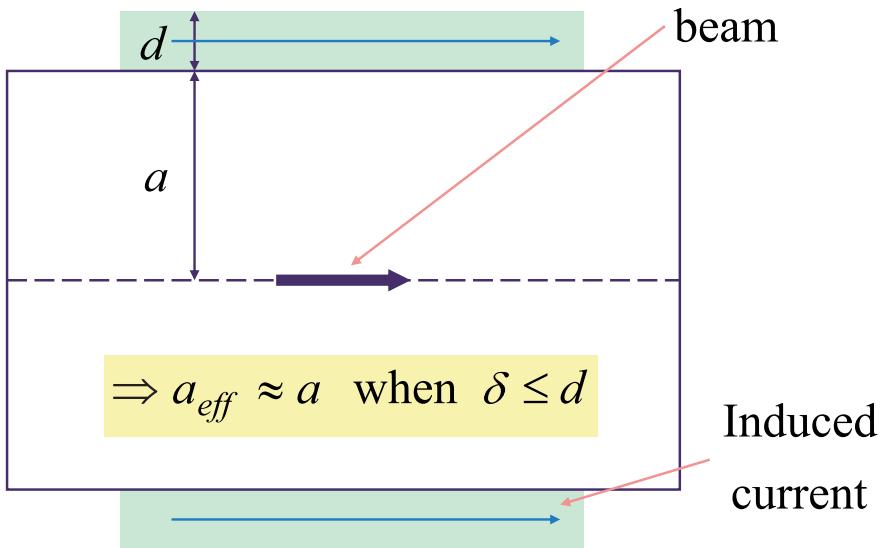
LHC Collimators: $b \ll d$



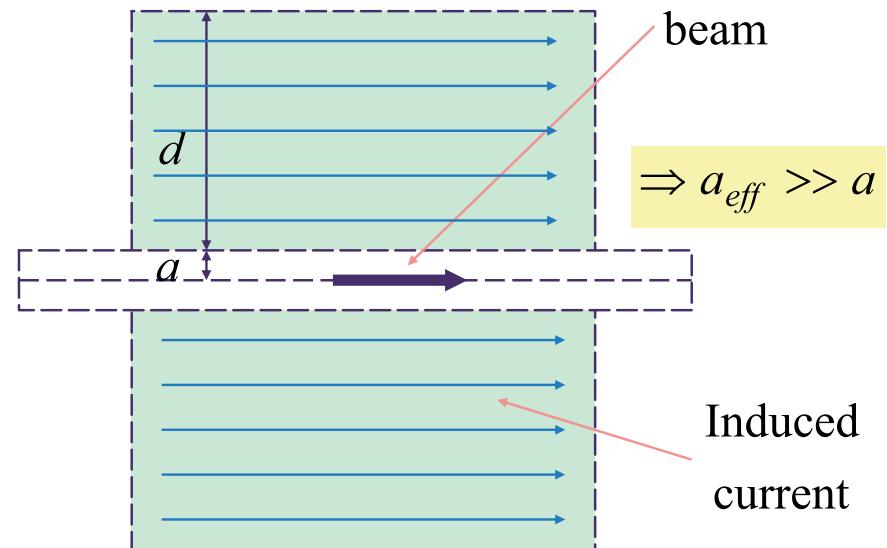
Resistive Wall Impedance Models



Usual regime : $d, \delta < a$



New regime : $d \gg a, \delta \leq d$



Standard parameter range: Wall thickness d is smaller than beam pipe radius a , and the skin depth δ for all frequencies under concern is smaller than d .

New regime: Wall thickness d is *larger* than beam pipe radius a , and the skin depth δ is in the order of d for low frequencies.



Resistive Wall Wake Function



$$Z_{m=1}^{\perp, \text{thick,ibp}}(\omega) = (1 + j \operatorname{sgn} \omega) \frac{c \mu_0 L}{2 \pi b^2} \frac{1}{-j + \operatorname{sgn} \omega \left(1 + b \sqrt{\frac{\sigma_c \mu_0}{2 \mu_r}} \sqrt{|\omega|} \right)}$$



Fourier Transform (not straightforward!)

$$W_{m=1}^{\perp, \text{thick,ibp}}(t) = \underbrace{+ \frac{cL}{\pi^{3/2} b^3} \sqrt{\frac{\mu_o \mu_r}{\sigma_c}} \cdot \frac{1}{\sqrt{|t|}}}_{\text{classic thick wall wake function}} \\ - \underbrace{\exp \left[\frac{4\mu_r}{b^2 \sigma_c \mu_0} |t| \right] \frac{2cL\mu_r}{b^4 \pi \sigma_c} \cdot \left(1 - \operatorname{Erf} \sqrt{\frac{4\mu_r}{b^2 \sigma_c \mu_0} |t|} \right)}_{\text{correction term due to inclusion of inductive bypass}}$$



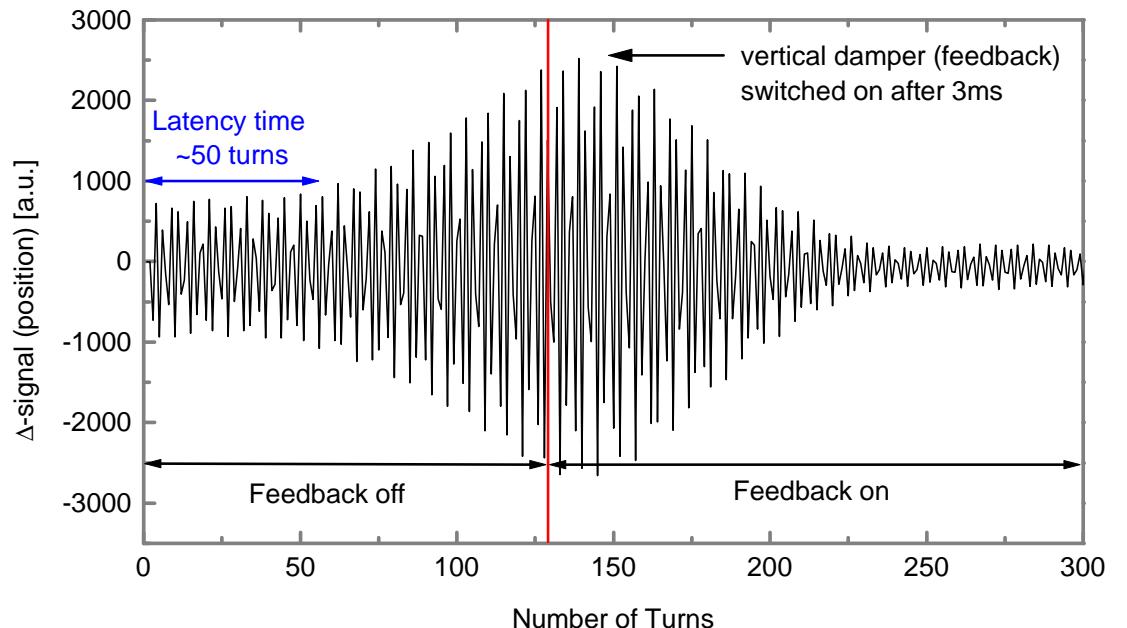
SPS Measurement & Simulation



Measurement Parameters

Fixed Target Beam	SPS @ inj.
Momentum p [GeV/c]	14
Revolution time $\tau_{\text{rev.}}$ [μs]	23.07
Tunes Q_H / Q_V	26.64 / 26.59
Gamma transition γ_T	23.2
Maximum # of batches	2
# of bunches per batch	2100
Bunch Intensity N_p	$4.8 \cdot 10^9$
Total Intensity $N_{p,tot.}$	$1.0 - 2.0 \cdot 10^{13}$
Batch spacing [ns]	1050
Bunch spacing [ns]	5
Full bunch length [ns]	4
Trans. emittance $\epsilon_{H,V}$ [μm]	$<10 / <7.5$
Long. emittance ϵ_L [eVs]	0.2

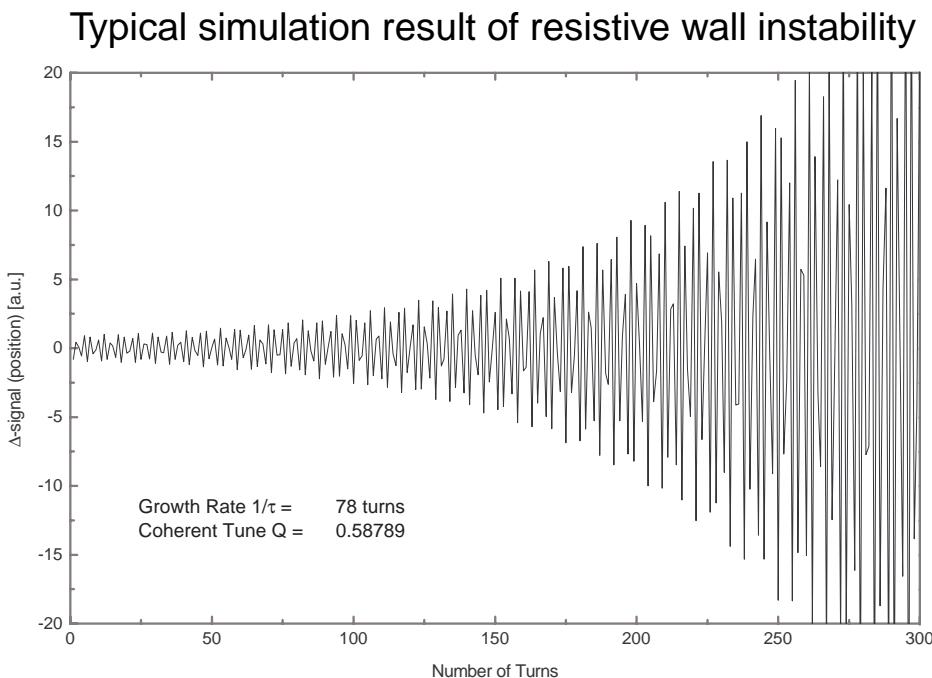
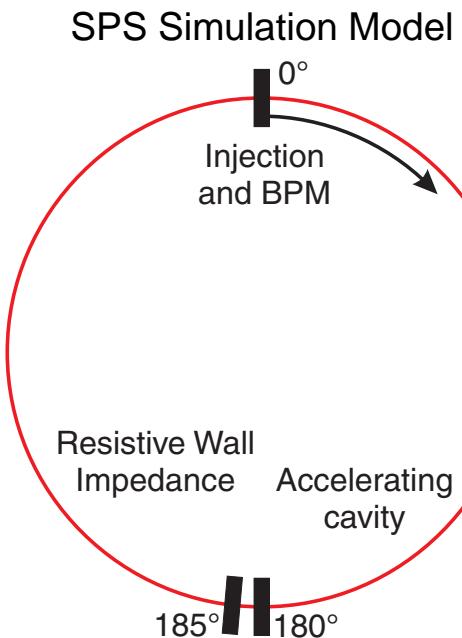
Typical vertical BPM readings for growth rate measurements



1 Batch (2100 bunches)	Growth rate $2\pi/\tau$ [turns]	Coherent tune Q
Vertical Plane	77 ± 4	0.5927 ± 0.0079
Horizontal Plane	183.5 ± 23.5	0.6180 ± 0.0029



SPS Measurement & Simulation



1 Batch (2100 bunches)	Growth rate $2\pi/\tau$ [turns]	Coherent tune Q
Vertical Plane	78 ± 2	0.58708 ± 0.0001
Horizontal Plane	150 ± 5	0.63854 ± 0.0001

SPS elliptic vacuum chamber geometry → Yokoya factors included!
Wake is not only dependent on exciting charge's offset, but also on witness particle offset:

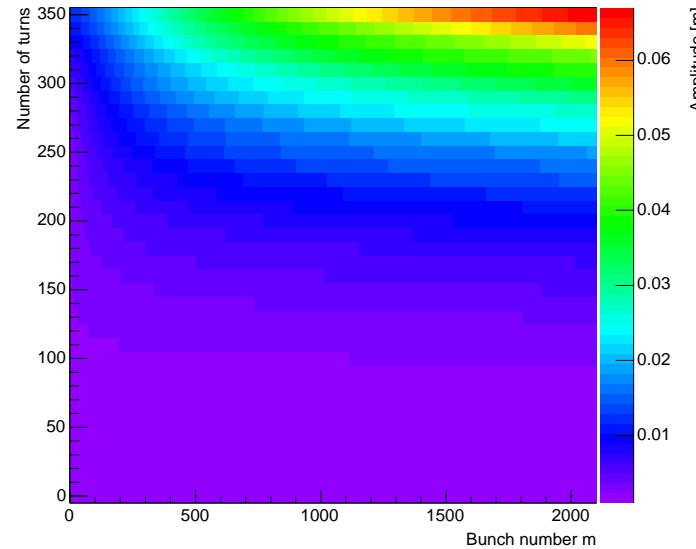
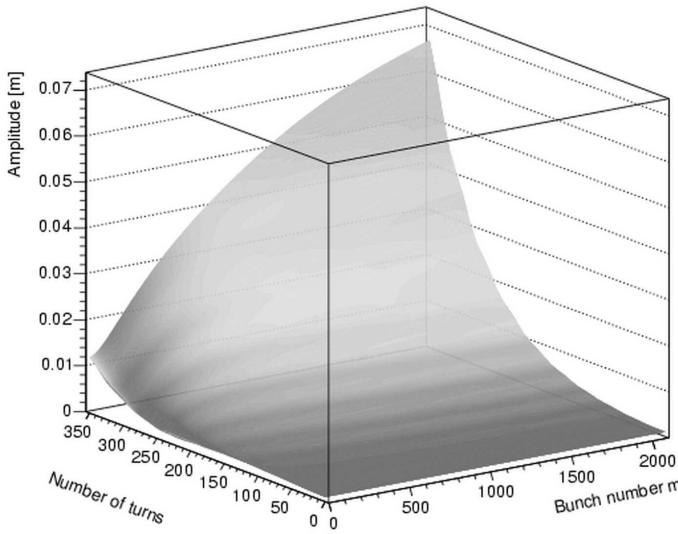
$$W_{\perp,x}^{\text{pot}}(x, y, \bar{x}, \bar{y}, s) \approx x W_{\perp}^{\text{pot}}(x, s) + \bar{x} W_{\perp}^{\text{pot}}(\bar{x}, s)$$

$$W_{\perp,y}^{\text{pot}}(x, y, \bar{x}, \bar{y}, s) \approx y W_{\perp}^{\text{pot}}(x, s) + \bar{y} W_{\perp}^{\text{pot}}(\bar{y}, s)$$

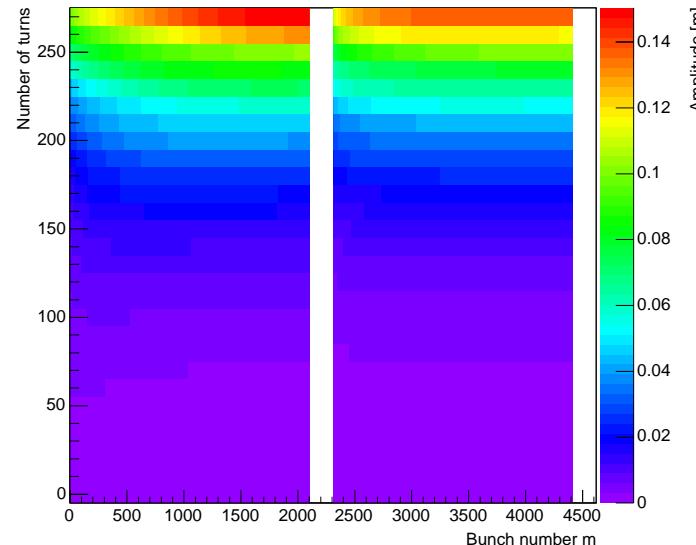
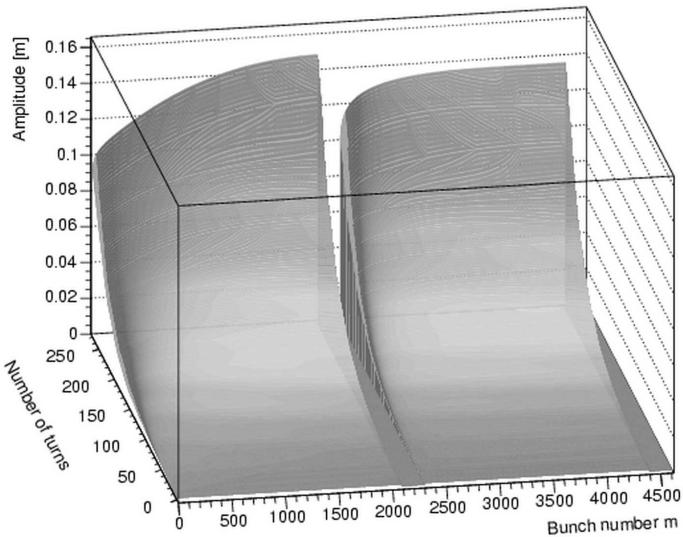
SPS Measurement & Simulation



Vertical Amplitude Growth, SPS FT beam, $n_b = 2100$ (1 batch)

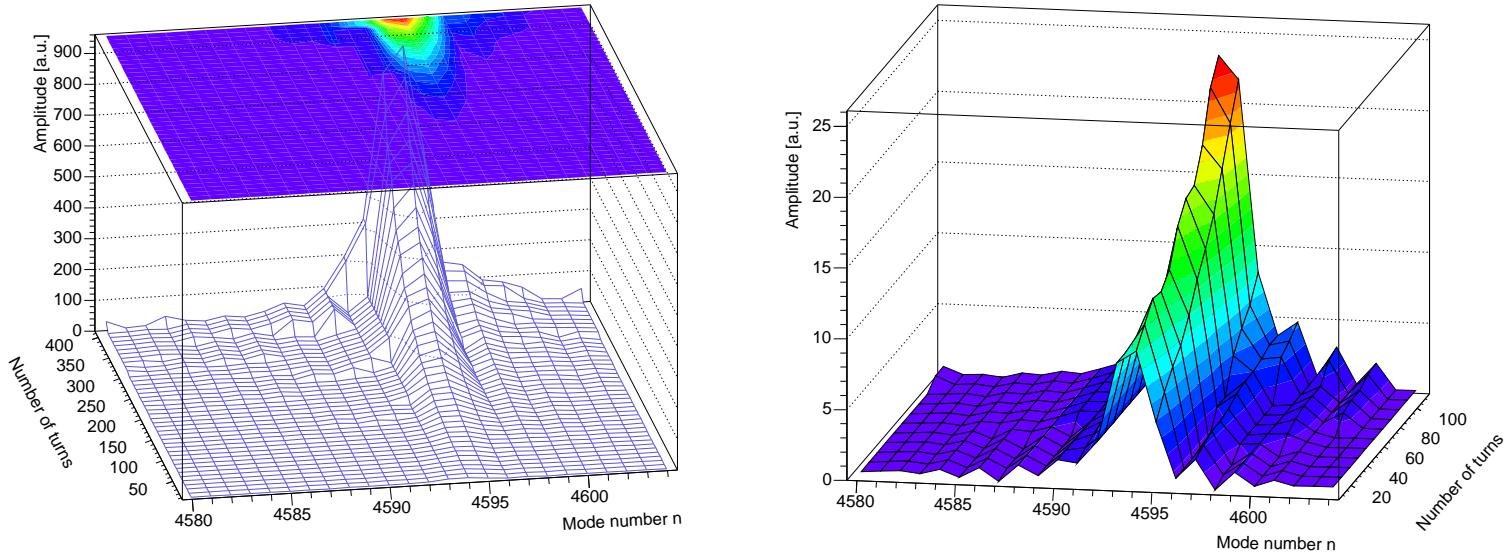


Vertical Amplitude Growth, SPS FT beam, $n_b = 4200$ (2 batches)

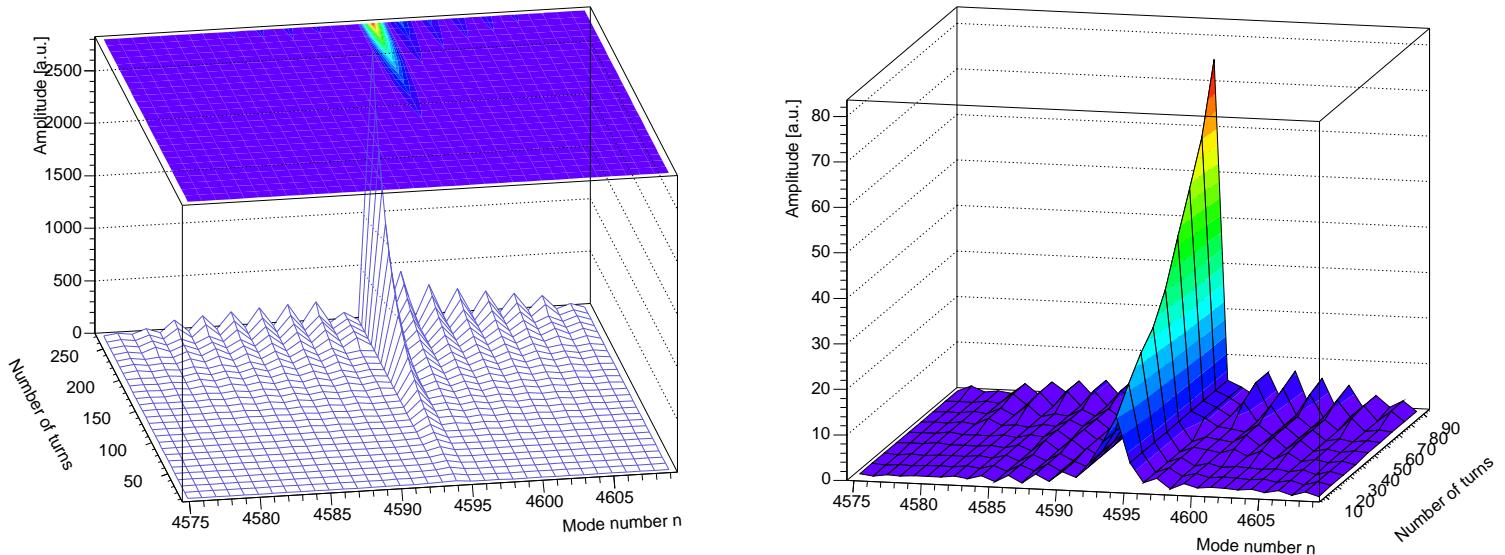


SPS Measurement & Simulation

Coupled Bunch Modes, SPS FT beam, $n_b = 2100$ (1 batch)



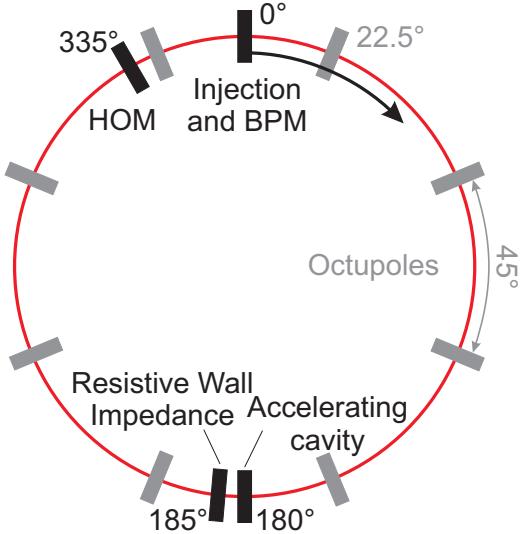
Coupled Bunch Modes, SPS FT beam, $n_b = 4200$ (2 batches)



Simulation of LHC



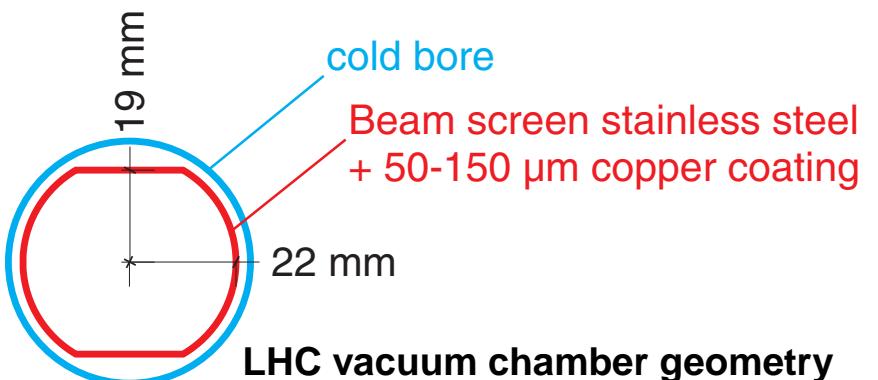
LHC Simulation Model



Parameter	Injection	Collision
Momentum p [GeV/c]	450	7000
Circumference C [m]		26658.883
Rev. frequency f_0 [Hz]		11245.5
Dipole field B [T]	0.535	8.33
Hor./Ver. Tune		64.28/59.31
Harmonic number h		35640
RF Frequency f_{RF} [MHz]		400.8
RF Voltage V_{RF} [MV]	8.0	16.0
Particles per bunch N_p [10^{11}]		1.15
Number of bunches n_b		2808
Bunch spacing τ_{sp} [ns]		25.0
Total # of particles N_{tot} [10^{14}]		3.23
Total DC beam current I [A]		0.582
Luminosity L [$\text{cm}^{-2}\text{s}^{-1}$]		$1.0 \cdot 10^{34}$

Impedances

- HOMs
(undamped modes of the 200MHz cavities)
- Resistive Wall
(Machine Resistance = mostly beam screen, collimators)



Non-Linearities

- Octupoles

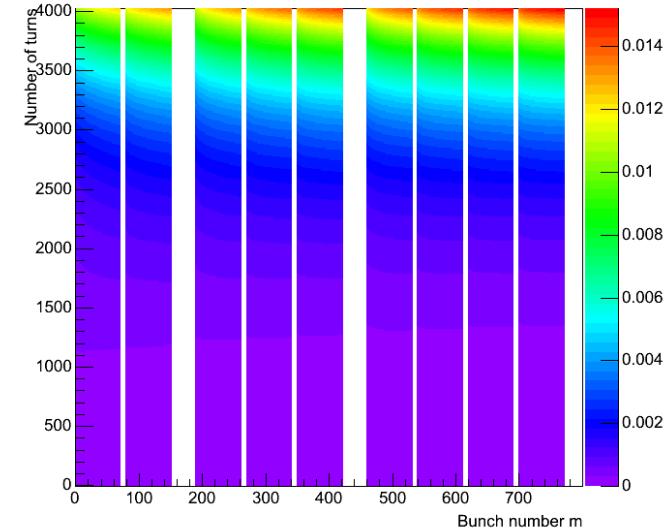
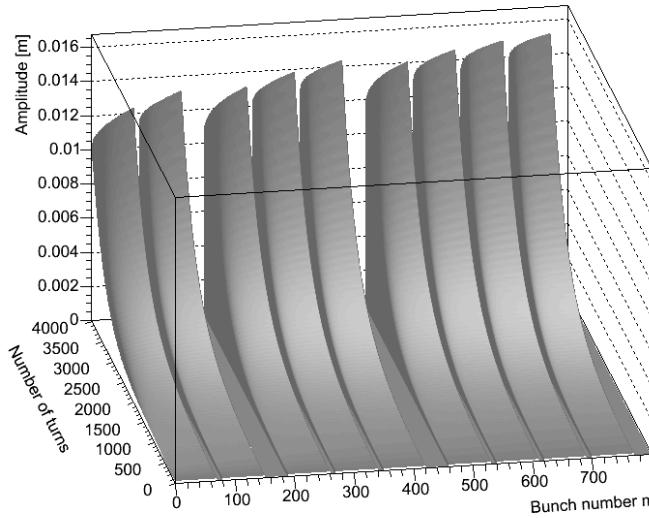
$$\delta x' = \frac{k_3 \cdot l}{3!} (x^3 - 3x y^2)$$

$$\delta y' = \frac{k_3 \cdot l}{3!} (3x^2 y - y^3)$$

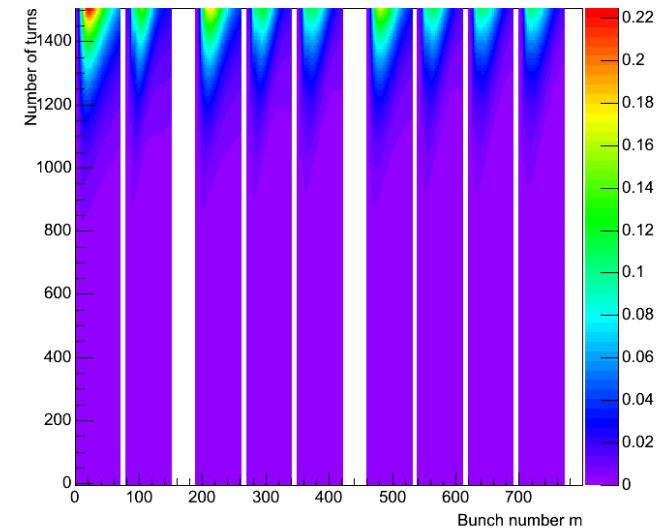
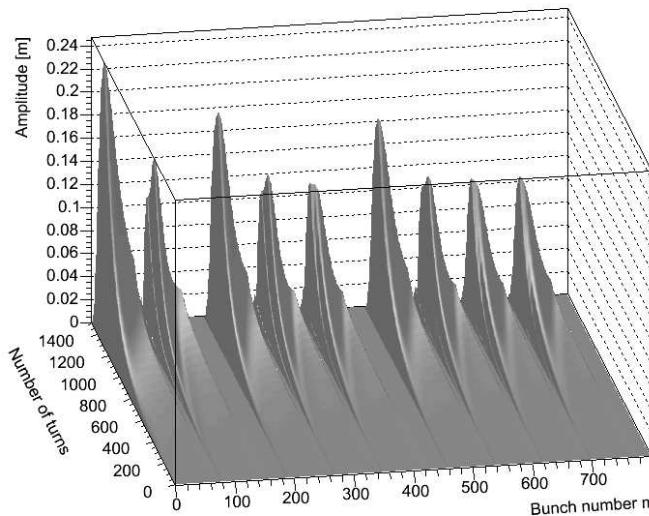
Simulation of LHC – Results



Amplitude growth of individual bunches vs. Turns
Machine Resistance only, LHC Injection Energy, nominal Intensity



Machine Resistance + Collimators, LHC Injection Energy, nominal Intensity

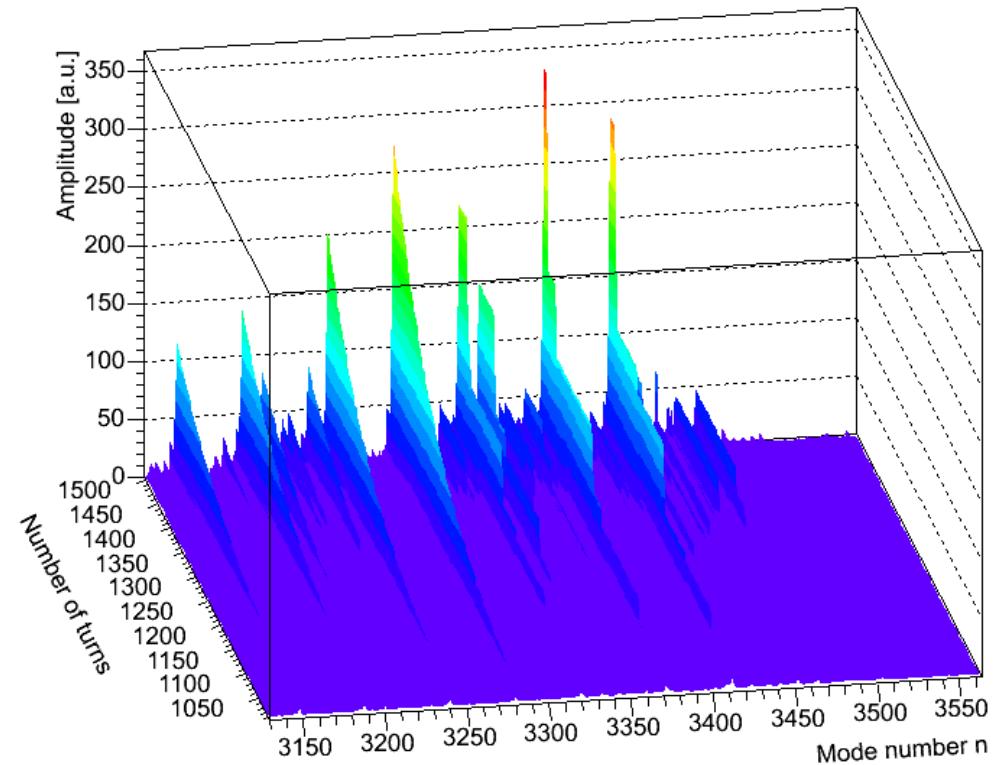
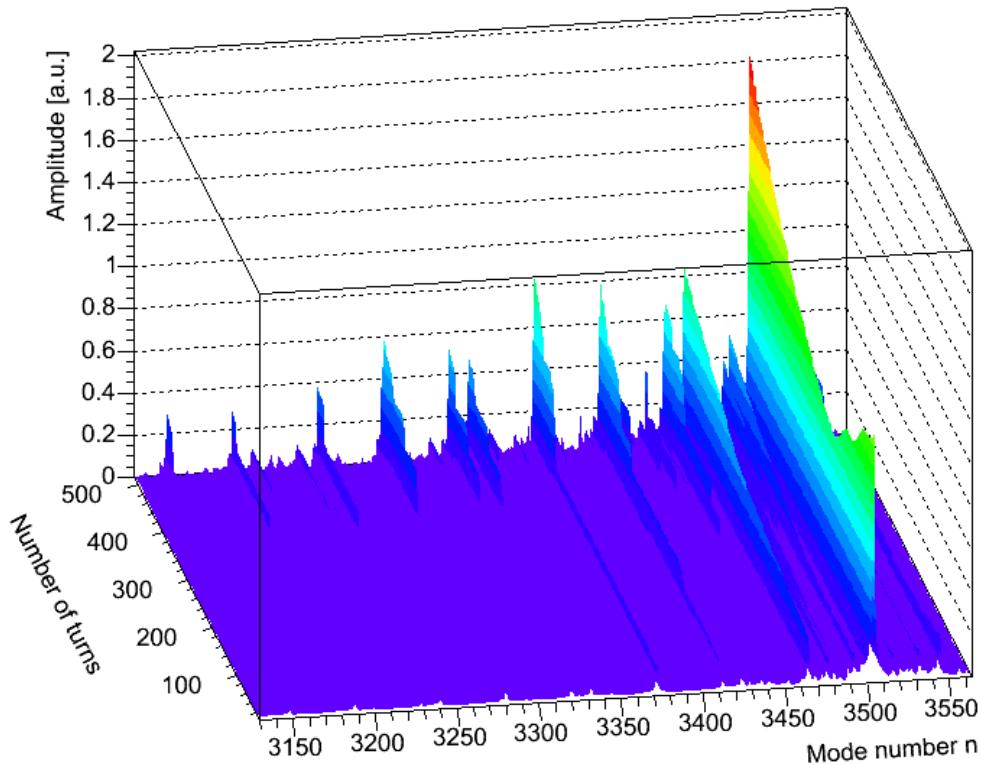


Simulation of LHC – Results



Coupled-bunch mode spectra vs. Turns at injection energy and nominal intensity

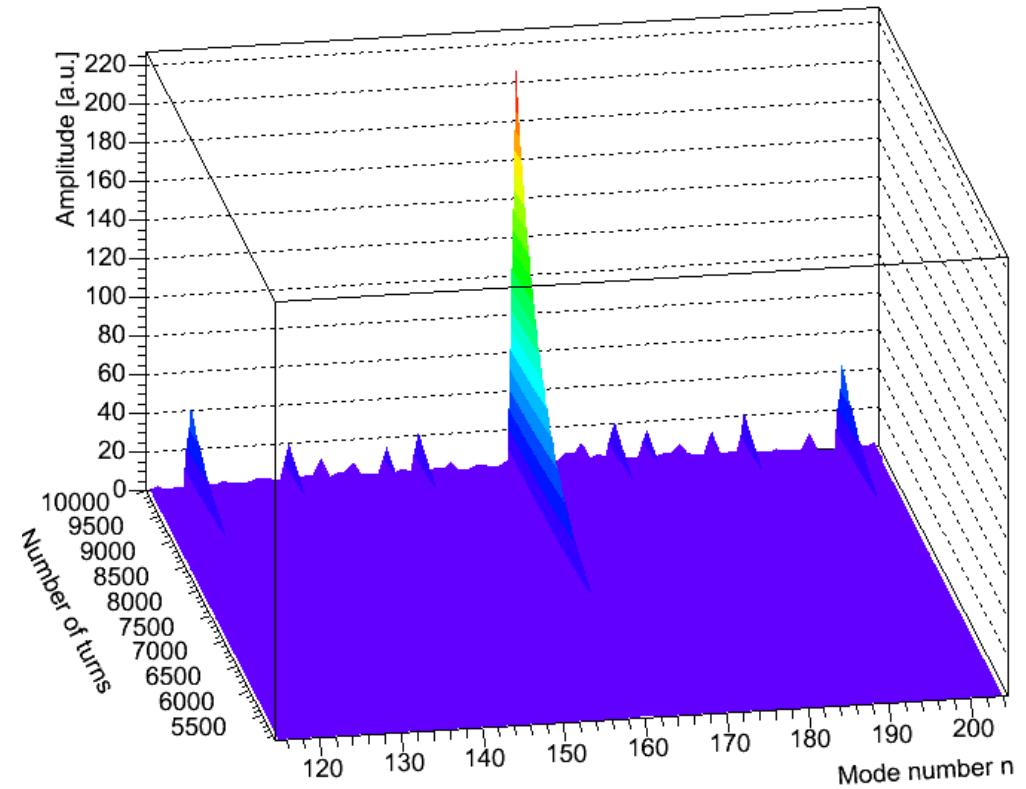
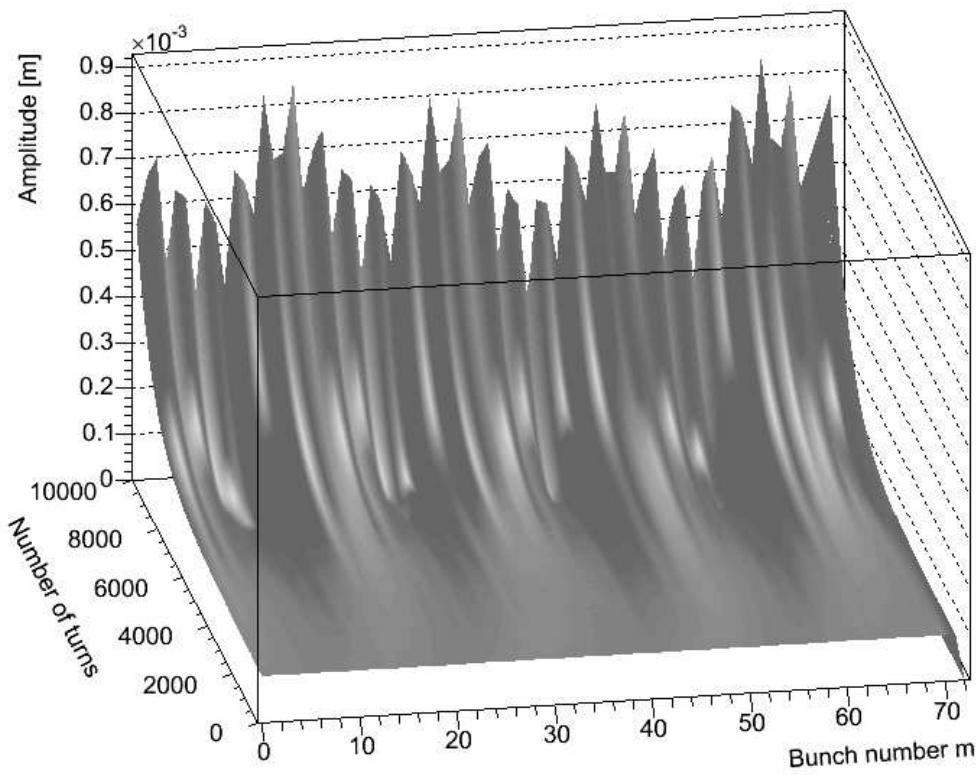
Machine Resistance + Collimators, LHC Injection Energy, nominal Intensity



Simulation of LHC – Results



Amplitude growth and coupled-bunch mode spectra vs. Turns
HOMs of 200MHz cavities, LHC Injection Energy, nominal Intensity

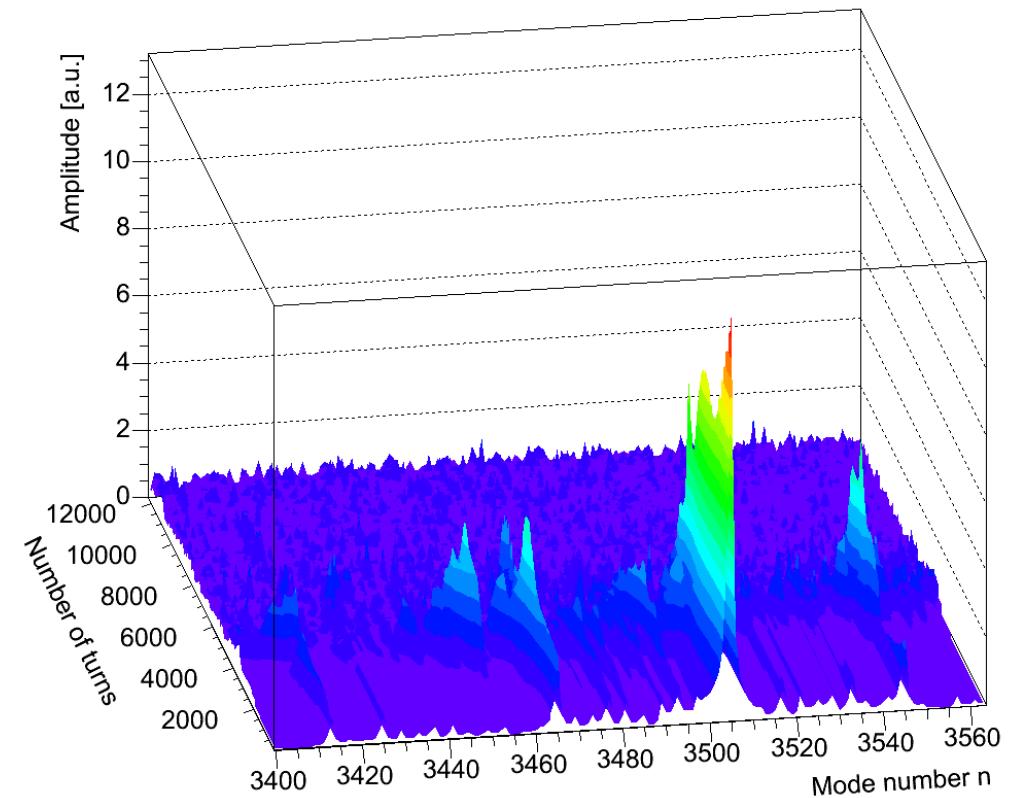
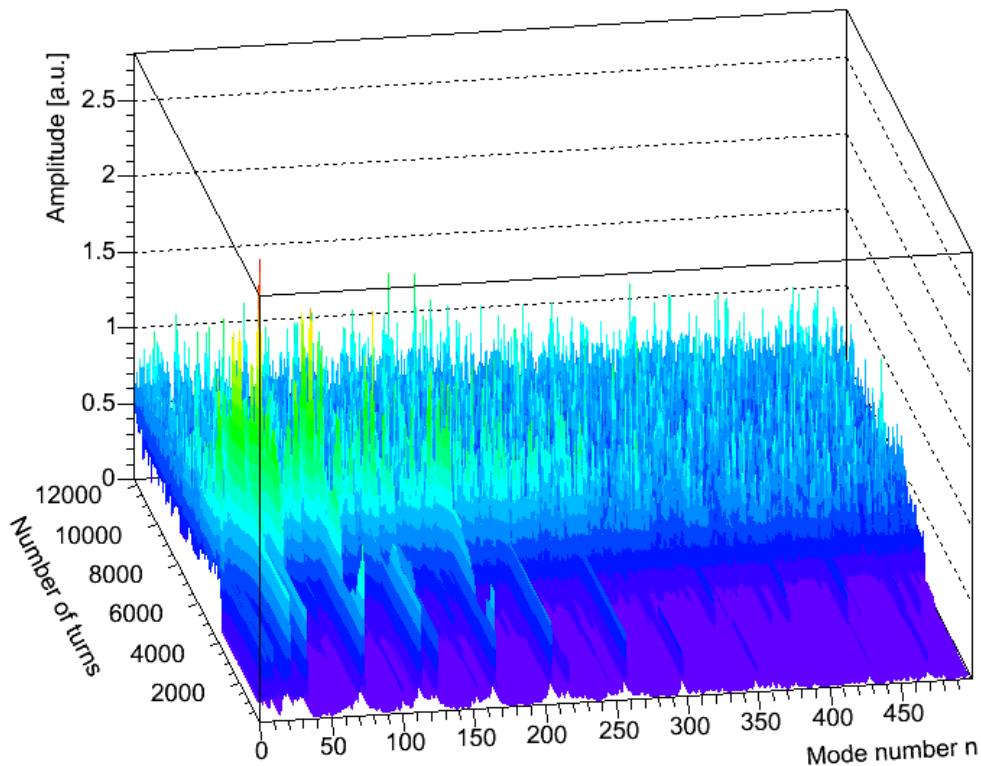


Simulation of LHC – Results



Coupled-bunch mode spectra vs. Turns

Machine Resistance + Collimators + HOMs and Octupoles
LHC Top Energy, ultimate Intensity



Non-linearities through octupoles → tune spread → Landau damping



Summary



- Simulation code *MultiTRISIM* developed
- Efficient implementation of wake summation via **FFT Convolution**
- Resistive wall impedance models in a new parameter regime $\delta_{\text{skin}}, d > b$
- Resistive wall **wake function with 'inductive bypass'** computed and used in simulation
- Code **benchmarked** with measurements in CERN SPS
- **Simulation of LHC.** Present octupole design should provide enough Landau damping to stabilize beam at top energy.

